Ad Soyad:

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Numara:

MAT 324 MATRİSLER TEORİSİ FİNAL SINAVI SORULARI

1. $A=\begin{bmatrix} 1 & 3 & 5 \\ 1 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ matrisinin adjoint (ek) matrisini bulunuz.

2. $A=\begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 2 & -1 & 3 \\ 1 & 2 & -1 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix}$ matrisi için detA=?

3. $A=\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$ matrisinin nilpotent matris olduğunu gösteriniz.

4. $\begin{cases} x+2y+z=0 \\ x+y=1 \\ 2x+z=2 \end{cases}$ lineer denklem sistemini çözünüz.

5. a) 2x2 tipinde bir elemanter matris yazıp tersini bulunuz.

b) $A=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ matrisinin öz (karakteristik) değer ve öz vektörlerini bulunuz.

NOT: Süre 75 dk

BAŞARILAR

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Cevap Anahtarı

1)

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 3 \\ 0 & 2 \end{vmatrix} = -2 \qquad A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 5 \\ 0 & 2 \end{vmatrix} = -6$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -5 \qquad A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 5 \\ -1 & 2 \end{vmatrix} = 7$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} = -1 \qquad A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = -3$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 5 \\ -1 & 3 \end{vmatrix} = 14$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 5 \\ 1 & 3 \end{vmatrix} = 2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} = -4$$

$$\tilde{A} = [A_{ij}]^t = \begin{bmatrix} -2 & -5 & -1 \\ -6 & 7 & -3 \\ 14 & 2 & -4 \end{bmatrix}^t = \begin{bmatrix} -2 & -6 & 14 \\ -5 & 7 & 2 \\ -1 & -3 & -4 \end{bmatrix}$$

2)

$$\begin{matrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{matrix}\begin{bmatrix} 1 & 0 & 2 & 0 \\ 1 & 2 & -1 & 3 \\ -1 & 2 & -1 & 0 \\ -1 & 1 & 1 & 1 \end{bmatrix} \; \varepsilon \sim \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 2 & -3 & 3 \\ 0 & 2 & -3 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

ε: $\alpha_2 \to \alpha_2 - \alpha_1$
$\alpha_3 \to \alpha_3 - \alpha_1$
$\alpha_4 \to \alpha_4 + \alpha_1$
determinant değişmez.

$$\begin{vmatrix} 1 & 0 & 2 & 0 \\ 0 & 2 & -3 & 3 \\ 0 & 2 & -3 & 0 \\ 0 & 1 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -3 & 3 \\ 2 & -3 & 0 \\ 1 & 3 & 1 \end{vmatrix} = -6 + 18 + 9 + 6 = 27$$

3- $A^p = 0$, p nilpotentlik derecesi.

$$A^2 = A \cdot A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4- $x+2y+z=0$

$x+y=1$

$2x+z=2$

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix} = 1-2-2 = -3 \neq 0$$

Cramer sistem.

$$x = \frac{\begin{vmatrix} 0 & 2 & 1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{vmatrix}}{-3} = \frac{-2-2}{-3} = \frac{4}{3}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{vmatrix}}{-3} = \frac{1+2-2}{-3} = -\frac{1}{3}$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix}}{-3} = \frac{2+4-4}{-3} = -\frac{2}{3}$$

5) a) Elemanter matris; Birim matrise birtek elemanter işlem uygulanarak elde edilir.

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{matrix}-\alpha_1\\ -\alpha_2\end{matrix} \qquad \varepsilon: \alpha_2 \to \alpha_2+\alpha_1 \text{ alınırsa}$$

$$E=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$ elemanter matristir.

Tersi $\varepsilon^{-1}: \alpha_2 \to \alpha_2-\alpha_1$ yazılır.

$$E^{-1}=\begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}$$ bulunur.

b) $$\begin{vmatrix}2-\lambda & 1\\ 1 & 2-\lambda\end{vmatrix}=0 \Rightarrow \lambda_1=1 \quad \lambda_2=3$$

$(A-I)(X)=0$ $$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$x+y=0 \quad x=-y$

$\lambda_1=1$ için öz vektör. $\alpha_1=t(1,-1)$, $t\in\mathbb{R}$

$(A-3I)(X)=0$ $$\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$-x+y=0 \quad x=y$

$\lambda_2=3$ için öz vektör $\alpha_2=(x,y)=t(1,1)$

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$t\in\mathbb{R}$